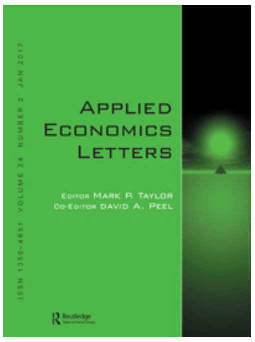

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Evolutionary dynamics of social tolerance in the economic interaction model with local social cost functions

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ABSTRACT

The evolutionary dynamics of social tolerance in a society with local social cost functions have been discussed. We show that, very different from the global function case studied previously, dynamics of social tolerance in a society with local social cost functions is integrable in phase space. The exact solution of the evolutionary dynamics in phase space is obtained, and the evolutionary trajectories are discussed by using the Kolmogorov–Arnold–Moser theorem. We show that the property of steady states is closely related to the group populations. We also discuss the necessary condition of the full tolerance steady state, and it is demonstrated that both global and local social cost functions have the same necessary condition of achieving full tolerance.

KEYWORDS

Evolutionary dynamics; social tolerance; local social cost function; economic interaction

JEL CLASSIFICATIONS

C73; D70

I. Introduction

Social tolerance, which is increasingly recognized as an important influence factor of economic growth, attracted more and more attention over the last decade (Akerlof and Kranton 2000; Berggren and Elinder 2012; Shi and Peng 2014). It has been suggested that social tolerance leads to many potentially important consequences, including technological and economic performance (Bomhoff and Lee 2012; Berggren and Nilsson 2013), population growth (Bjørnskov 2004) and social development (Bjørnskov 2008). The discussion on tolerance at the individual level reveals that economic reasoning can offer original and unique insights into the determinants of tolerance (Corneo and Jeanne 2009), and many social phenomena related to tolerance can be explained by economic models (Garofalo, Di Dio, and Correani 2010; Muldoon, Borgida, and Cuffaro 2012).

Recently, as a natural continuation of economic studies on fundamentalism (Darity, Mason, and Stewart 2006; Epstein and Gang 2007), an evolutionary economic interactions model of social tolerance has been studied (Cerqueti, Correani, and Garofalo 2013) by introducing a social cost function $c_i(x_1, x_2)$, where x_1 and x_2 are the share of tolerant agents in group 1 and group 2, respectively. The social cost function obeys the following properties:

(1) $\frac{\partial c_i(x_1,x_2)}{\partial x_1} < 0$ and $\frac{\partial c_i(x_1,x_2)}{\partial x_2} < 0$;
(2) $\frac{\partial^2 c_1(x_1,x_2)}{\partial x_1^2} \geq 0$ and $\frac{\partial^2 c_2(x_1,x_2)}{\partial x_2^2} \geq 0$;
(3) $\frac{\partial^2 c_1(x_1,x_2)}{\partial x_1 \partial x_2} \leq 0$ and $\frac{\partial^2 c_2(x_1,x_2)}{\partial x_2 \partial x_1} \leq 0$;
(4) If $x_1 = x_2 = 0$, then $c_i(x_1, x_2) = 0$.

The first property states that the individual cost increases when the share of intolerant people in group 1 or group 2 increases, which is a global dependence. We note that the social cost function may be local in some cases, and the individual cost depends only on the share of intolerant people in his own group. The local social cost function obeys the following properties:

(1) $\frac{\partial c_1(x_1,x_2)}{\partial x_1} < 0$ and $\frac{\partial c_2(x_1,x_2)}{\partial x_2} < 0$;
(2) $\frac{\partial^2 c_1(x_1,x_2)}{\partial x_1^2} \geq 0$ and $\frac{\partial^2 c_2(x_1,x_2)}{\partial x_2^2} \geq 0$;
(3) $\frac{\partial^2 c_1(x_1,x_2)}{\partial x_1 \partial x_2} \leq 0$ and $\frac{\partial^2 c_2(x_1,x_2)}{\partial x_2 \partial x_1} \leq 0$;
(4) $c_1(x_1, x_2) = 0$ if $x_1 = 0$, while $c_2(x_1, x_2) = 0$ if $x_2 = 0$.

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Such local social cost function can find application in many realistic examples, such as spatially or politically separated groups. So it is interesting to study the dynamics of tolerance in the evolutionary economic interactions model with local social cost functions. Here, we consider a simple local social cost function $c_i(x_1, x_2) = \beta(1 - x_i)$ instead of the global function $c_i(x_1, x_2) = \beta(1 - x_1 x_2)$ used by Cerqueti, Correani, and Garofalo (2013). We show that the evolutionary dynamics of social tolerance under local social cost function is integrable in phase space and interactions between individuals in a society with local social cost functions sometimes lead to rich behaviours which are different from the global function case.

II. Evolutionary model of tolerance dynamics

We use an evolutionary game model of social tolerance similar to Cerqueti, Correani, and Garofalo (2013). We consider that there are two differentiated groups of economic agents with the total population N. The number of members of each group, which recorded as N_1 and N_2, is assumed to be large enough and changeless with time. Each individual can be tolerant or intolerant towards the agents of another group. We indicate that x_i and $\hat{x}_i$ be the share of tolerant and intolerant agents in group i, respectively, thus $x_i + \hat{x}_i = 1$ and $x_i, \hat{x}_i \in [0, 1]$, for each $i = 1, 2$.

The evolutionary dynamics of social tolerance can be modelled by the theory of replicators in which two agents interact after being randomly matched, producing aggregate wealth $R_{ij} = R_{ji}$, which depend on the capital contribution of these agents. The capital contribution of agents in group i is denoted by k_i, and the relative capital contribution that agents in group i interact with agents in group j is defined as $\delta_{ij} \equiv k_i/(k_i + k_j)$, and determines the shares of the aggregate wealth: the agent in group i shares $\delta_{ij} R_{ij}$ when he interact with the agent in group j, which yields $\delta_{ij} + \delta_{ji} = 1$, $\forall i, j = 1, 2$. The social tolerance influences the net gain of each agent in the following cases:

(1) For the case that the two agents are of the same group, whether tolerance or not, $\delta_{ii} = 1/2$ and each agent obtains $R_{ii}/2$;
(2) For the case that the two agents are of different group and both tolerance, they suffer a psychological cost and a social cost with the exception of $\delta_{ij} R_{ij}$. The psychological cost in terms of loss of identity is chosen to be $\alpha_i = R_{ii}/2$(Akerlof and Kranton 2000; Cerqueti, Correani, and Garofalo 2013), while the social cost is chosen to be local here. Here, we consider a simple local social cost function $c_i(x_1, x_2) = \beta(1 - x_i)$by comparison to the global function $c_i(x_1, x_2) = \beta(1 - x_1 x_2)$ used by Cerqueti, Correani, and Garofalo (2013). The parameter β is greater than zero, and a higher β leads to a higher social costs, so the parameter β describes the social reaction of intolerant agents in group i adverse to the agents of the group j.
(3) For the case that the two agents are of different group, there is no wealth produced if any of them is intolerant.

The evolutionary dynamics of social tolerance can be modelled by the theory of replicators, and the evolution of tolerant population in group i can be described by

$$\dot{x}_i = x_i \hat{x}_i (E[x_i] - E[\hat{x}_i]), \tag{1}$$

where $E[x_i]$ and $E[\hat{x}_i]$ are the expected net gain of tolerant and intolerant individuals in group i, respectively. Using the probabilities for the match between agents, these expected net gains can be calculated as follows:

$$\begin{aligned} E[x_1] &= P_{x_1 x_1} R_{11}/2 + P_{x_1 \hat{x}_1} R_{11}/2 + [\delta_{12} R_{12} - R_{11}/2 \\ &\quad - \beta(1 - x_1)] P_{x_1 x_2}, \\ E[x_2] &= P_{x_2 x_2} R_{22}/2 + P_{x_2 \hat{x}_2} R_{22}/2 + [\delta_{21} R_{21} - R_{22}/2 \\ &\quad - \beta(1 - x_2)] P_{x_2 x_1} \\ E[\hat{x}_1] &= P_{\hat{x}_1 x_1} R_{11}/2 + P_{\hat{x}_1 \hat{x}_1} R_{11}/2, \\ E[\hat{x}_2] &= P_{\hat{x}_2 x_2} R_{22}/2 + P_{\hat{x}_2 \hat{x}_2} R_{22}/2, \end{aligned} \tag{2}$$

with $P_{x_i x_j}$(or $P_{x_i \hat{x}_j}$) the probability for a tolerant agent of group i matches a tolerant (or intolerant) agent of group j. Considering the random match, in which all agents have the same probability to be selected, we can obtain $P_{x_i x_j}$, $P_{x_i \hat{x}_j}$, $P_{\hat{x}_i x_j}$ and $P_{\hat{x}_i \hat{x}_j}$ as follows:

$$
\begin{aligned}
&P_{x_1x_1}=\frac{x_1N_1-1}{N-1}, \quad P_{x_1\hat{x}_1}=\frac{\hat{x}_1N_1}{N-1}, \quad P_{x_1x_2}=\frac{x_2N_2}{N-1}, \quad P_{x_1\hat{x}_2}=\frac{\hat{x}_2N_2}{N-1},\\
&P_{\hat{x}_1x_1}=\frac{x_1N_1}{N-1}, \quad P_{\hat{x}_1\hat{x}_1}=\frac{\hat{x}_1N_1-1}{N-1}, \quad P_{\hat{x}_1x_2}=\frac{x_2N_2}{N-1}, \quad P_{\hat{x}_1\hat{x}_2}=\frac{\hat{x}_2N_2}{N-1},\\
&P_{x_2x_1}=\frac{x_1N_1}{N-1}, \quad P_{x_2\hat{x}_1}=\frac{\hat{x}_1N_1}{N-1}, \quad P_{x_2x_2}=\frac{x_2N_2-1}{N-1}, \quad P_{x_2\hat{x}_2}=\frac{\hat{x}_2N_2}{N-1},\\
&P_{\hat{x}_2x_1}=\frac{x_1N_1}{N-1}, \quad P_{\hat{x}_2\hat{x}_1}=\frac{\hat{x}_1N_1}{N-1}, \quad P_{\hat{x}_2x_2}=\frac{x_2N_2}{N-1}, \quad P_{\hat{x}_2\hat{x}_2}=\frac{\hat{x}_2N_2-1}{N-1},
\end{aligned}
$$

Given the above probabilities, the motion of tolerant population with respect to time will be then modelled by the following differential equations:

$$
\begin{aligned}
\dot{x}_1&=\frac{x_1\hat{x}_1}{N-1}[\delta_{12}R_{12}-R_{11}/2-\beta(1-x_1)]x_2N_2,\\
\dot{x}_2&=\frac{x_2\hat{x}_2}{N-1}[\delta_{21}R_{21}-R_{22}/2-\beta(1-x_2)]x_1N_1.
\end{aligned} \tag{3}
$$

These equations give a complete description of the evolutionary dynamics of social tolerance. The steady states of Equation 3 are as follows:

$P_1=(1,1),\ P_2=(\xi_1,0),\ P_3=(0,\xi_2),\ P_4=(0,0),$

$P_5=(1,1-\Omega_2),\ P_6=(1-\Omega_1,1),\ P_7=(1-\Omega_1,1-\Omega_2),$

where $\Omega_1=[\delta_{12}R_{12}-R_{11}/2]/\beta$, $\Omega_2=[\delta_{21}R_{21}-R_{22}/2]/\beta$ and $\xi_1,\xi_2\in(0,1]$ are arbitrary constants. These steady states have precise economic and social meanings. For example, P_1 is the full tolerance steady state, P_2 and P_3 depict situations that one group is wholly populated by intolerant agents while another group is arbitrary tolerant. P_4 is the full intolerance steady state. Economic and social meanings for steady states P_1–P_4 are relatively simple, while P_5–P_7 are more complex. Among them, P_5 exists when $\Omega_1\in(0,1)$, which requires $0<\delta_{12}R_{12}-R_{11}/2<\beta$, while P_6 exists when, $\Omega_2\in(0,1)$ which requires $0<\delta_{21}R_{21}-R_{22}/2<\beta$. So steady states P_5–P_7 are determined by the distribution of aggregate wealth. We note that some of the steady states, such as P_5–P_7, are different from the global function case.

III. Solutions in the phase space

The differential equations that describe the evolutionary dynamics of social tolerance under global social cost function (Cerqueti, Correani, and Garofalo 2013) are non-integrable in general; however, Equation 3 which describes the evolutionary dynamics of social tolerance under local social cost function is integrable. In what follows, we give a description of the derivation of the solution in phase space.

Proposition 1. The solution of evolutionary social tolerance under local social cost function in phase space is

$$
\left|\frac{x_1-1}{x_1-(1-\Omega_1)}\right|=C\left|\frac{x_2-1}{x_2-(1-\Omega_2)}\right|^{\gamma}, \tag{4}
$$

where C is an arbitrary constant depends on initial values, and $\gamma=\frac{N_2}{N_1}\frac{\Omega_1}{\Omega_2}$ determines the exponential relationship between x_1 and x_2.

Proof. In the phase space, Equation 3 becomes:

$$
\frac{\mathrm{d}x_1}{\mathrm{d}x_2}=\frac{\hat{x}_1N_2[x_1-(1-\Omega_1)]}{\hat{x}_2N_1[x_2-(1-\Omega_2)]}. \tag{5}
$$

This equation can be solved exactly and the solution is

$$
\ln\left|\frac{x_1-1}{x_1-(1-\Omega_1)}\right|=\ln C\left|\frac{x_2-1}{x_2-(1-\Omega_2)}\right|^{\frac{N_2\Omega_1}{N_1\Omega_2}} \tag{6}
$$

After some simplification we obtain Equation 4.

From the solution, the steady states of the evolutionary dynamics can be easily identified, and the evolutionary trajectory can be exactly analysed. According to the Kolmogorov–Arnold–Moser theorem, the evolutionary trajectories are confined to closed tori that translate into regular closed orbits on Poincare sections for a regular oscillation in the present case.

The solution in the range of $x_1,x_2\in[0.8,1]$ is shown in Fig. 1. We can clearly identify the steady states P_1 and P_7. It is shown that the steady state P_7 is unstable, and the evolutionary dynamics near the

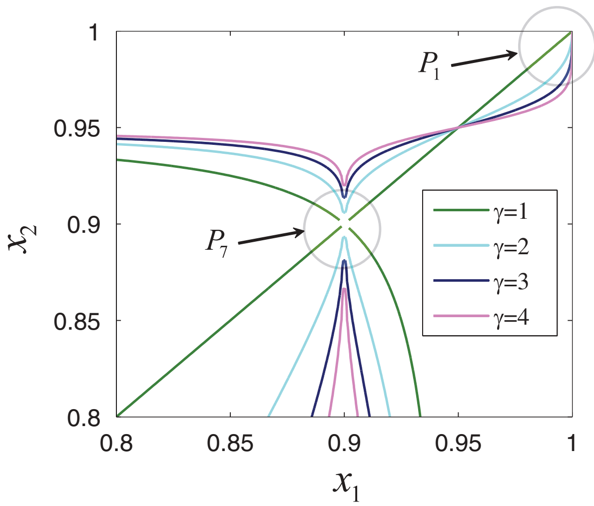

Figure 1. Exact solutions of Equation 3 in phase space with different γ. We use $\Omega_1 = \Omega_2 = 0.1$.

steady state P_7 is sensitively dependent on γ, which is determined by population numbers of each group and the distribution of aggregate wealth.

IV. Full tolerance steady state

The full tolerance steady state $(1, 1)$ is of particular interest. In this section, we focus our attention on discussing the necessary condition of the full tolerance steady state $(1, 1)$. Other steady states can be discussed similarly.

Proposition 2. The necessary condition of achieving full tolerance is

$$R_{11} + R_{22} - 2R_{12} < 0. \tag{7}$$

Proof. To verify the stability of steady state $(1, 1)$, we write $x_1 = 1 - \delta x_1$ and $x_2 = 1 - \delta x_2$ with $0 \leq \delta x_1 \ll 1$ and $0 \leq \delta x_2 \ll 1$, thus Equation 3 can be reduced to

$$\delta\dot{x}_1 = -\frac{\delta x_1}{N-1}(\delta_{12}R_{12} - R_{11}/2)N_2, \tag{8}$$

$$\delta\dot{x}_2 = -\frac{\delta x_2}{N-1}(\delta_{21}R_{21} - R_{22}/2)N_1, \tag{9}$$

where higher-order small quantities of δx_1 and δx_2, such as δx_1^2, are ignored, and the solutions of Equations 8 and 9 are

$$\delta x_1 = \exp\left[-\frac{N_2}{N-1}(\delta_{12}R_{12} - R_{11}/2)t\right], \tag{10}$$

$$\delta x_2 = \exp\left[-\frac{N_1}{N-1}(\delta_{21}R_{21} - R_{22}/2)t\right], \tag{11}$$

So the necessary conditions of tolerance spreads in group i which ensure $\delta x_i \to 0$ for $t \to \infty$ are

$$\delta_{12}R_{12} - R_{11}/2 > 0, \tag{12}$$

$$\delta_{21}R_{21} - R_{22}/2 > 0. \tag{13}$$

After some simplification we obtain Inequality 7.

This necessary condition of achieving full tolerance is exactly the result obtained by Cerqueti, Correani, and Garofalo (2013). The economics meaning is that tolerance is impossible if $R_{11} + R_{22} - 2R_{12} > 0$ where R_{12} is not sufficiently high to produce a tendency of mixed interaction in economic incentive. Our result shows that local social cost functions should not change the necessary condition of achieving full tolerance. In fact, sufficient conditions of achieving full tolerance are in the same situation.

V. Conclusions

We discuss the dynamics of social tolerance in a society with local social cost functions, which is integrable in phase space and can be discussed more easily than the model with global social cost functions (Cerqueti, Correani, and Garofalo 2013). We obtain the exact solution of the evolutionary dynamics in phase space, and discuss the evolutionary trajectories by using the Kolmogorov–Arnold–Moser theorem. Very different from the global function case studied previously, dynamics of social tolerance in a society with local social cost functions is closely related to the group populations. We also show that both global and local social cost functions have the same necessary condition of achieving full tolerance.

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Disclosure statement

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